

## CPA: Concrete to Pictorial to Abstract

### Basic Overview

Every lesson in the *Math in Focus* curriculum is built on the Concrete to Pictorial to Abstract method of mastery. Research shows that children master mathematical concepts most effectively using this sequence (see links below). Students begin exploration of a concept by Concrete manipulatives that they can handle and rearrange. Once they understand the idea using objects, they move to a Pictorial representation. These pictures further develop their understanding. Now they are ready to move to a problem where they must come up with strategies to find possible solutions.

### More about CPA

CPA Explanation, Research, Background and Examples:

<http://www.loganschools.org/mathframework/CPA.pdf>

A background of the research and reasoning behind *Math in Focus* (for CPA specifics, see pp. 9 & 13 (pages 11 & 15 of the pdf))

[http://www.hmheducation.com/singaporemath/pdf/MIF\\_Research\\_HR.pdf](http://www.hmheducation.com/singaporemath/pdf/MIF_Research_HR.pdf)

Ideas for “Math Play” at home

<http://www.hmheducation.com/singaporemath/helping-your-child.php>

## Number Bonds

### Basic Overview

As students advance their number sense, they begin to see numbers as more than just a word. They begin to break numbers into parts of the whole, thereby increasing what can be done with and to that number. This technique is known in *Math in Focus* as Number Bonds. For example, when a Singapore math student sees the number 8, their mind automatically sees  $6 + 2$ ,  $3 + 5$ ,  $7 + 1$ , and double 4. This is helpful when adding, subtracting, multiplying and dividing the number 8. When a traditional math student is given the problem  $8 + 6$ , they must either count or memorize the answer. Singapore math students realize that 8 must bond with 2 to make 10 (a number that is easy to add to and subtract from), so they break the number 6 into its bond of  $2 + 4$ , pairing the 2 with the 8 to make 10. It is then easy for them to add the remaining 4 to the 10 to make 14. When a child (or adult) begins to see numbers in this way, once complex and overwhelming problems become simple and straightforward.

### More about Number Bonds

Understanding Number Bonds

<http://letsplaymath.net/2007/01/13/number-bonds-better-understanding/>

Online games to practice Number Bonds

Ten Frames

<http://illuminations.nctm.org/activitydetail.aspx?id=75>

Save the Whale: Number Bonds to Ten

[http://www.ictgames.com/save\\_the\\_whale\\_v4.html](http://www.ictgames.com/save_the_whale_v4.html)

Number Bond Machines

<http://www.amblesideprimary.com/ambleweb/mentalmaths/numberbond.html>

Alien Attack: Number Bonds with Fractions and Decimals

<http://www.mathplayground.com/PartPartWhole.html>

## Place Value

### Basic Overview

The comprehension of Place Value allows students to understand the *why* behind many mathematical algorithms. Students move from understanding ones and tens values, to being able to comprehend systems of tens to the millions and thousandths places. Addition, subtraction, multiplication, division fractions and decimals are explored through the scope of this imperative concept of place value.

### More about Place Value

Place Value in Singapore Math (esp. p. 1 &7)

<http://www.hmheducation.com/singaporemath/pdf/PlaceValue.pdf>

## Model Drawing

### Basic Overview

Model Drawing is a method of representing knowns and unknowns of a problem by bars of differing lengths. Its systematic process, once mastered, gives students of all mathematical talent the ability to work through almost any word problem. The beauty of the bar model is that it can give a visual representation of large and small numbers. It makes sense of addition, subtraction, multiplication, and division. Fractions and ratios are easy to see and understand. To see the steps of creating a Bar Model, go to <http://heritagechristian.info/elementary/math-in-focus/> and click on the link "HCA Bar Models."

### More about Model Drawing

*Math in Focus* Bar Modeling

<http://www.hmheducation.com/singaporemath/pdf/ModelDrawing.pdf>

Problem Solving in Singapore Math

<http://www.hmheducation.com/singaporemath/pdf/MIFProbSolving.pdf>

Online games to practice Model Drawing:

Interactive Drawing Tool

<http://www.thinkingblocks.com/>

Matching Models to Word Problems

<http://www.thesingaporemaths.com/index.html>

iPad app for Model Drawing (\$9.99)

<http://www.hmheducation.com/singaporemath/ipad.php>

## Other Resources

Problem Solving in Singapore Math

<http://www.hmheducation.com/singaporemath/pdf/MIFProbSolving.pdf>

Theory behind *Math in Focus*

<http://www.hmhededucation.com/singaporemath/pdf/MIFTheUnderpinningConcept.pdf>

#### Research and Statistics

“Study Finds Houghton Mifflin Harcourt’s Singapore Math Program Raises Student Achievement in the U.S.”

<http://www.businesswire.com/news/home/20110118006227/en/Study-Finds-Houghton-Mifflin-Harcourt%E2%80%99s-Singapore-Math>

*Math in Focus* Research Base

[http://www.hmhededucation.com/singaporemath/pdf/MIF\\_Research\\_HR.pdf](http://www.hmhededucation.com/singaporemath/pdf/MIF_Research_HR.pdf)

International Comparison Studies

<http://www.hmhededucation.com/singaporemath/pdf/IntlComparisonStudies.pdf>

Analysis of State Test Scores in Old Bridge, NJ

<http://www.hmhededucation.com/singaporemath/pdf/StateTestScoreAnalysisNJ.pdf>

Singapore Math Statistics

<http://isingaporemath.com/index.php/statistics/singapore-math-statistics>

#### Other Math Games

Factor Trees and Prime Factorization

<http://mathplayground.com/factortrees.html>

Timez Attack: Video Game for practicing math facts

<http://www.bigbrainz.com/>

Interactive Fraction Tools

<http://www.conceptuamath.com/fractions.html>