## ${ }^{66}$ CPA ${ }^{99}$

## Concrete

 --Objects--Touch and move
Pictorial
--Pictures of objects
--On paper

## Abstract

--Reason through strategies to
solve

## Place Value

## 3,726

## $3,000+700+20+6$

$$
\begin{array}{r}
27 \\
+\quad 35 \\
\hline
\end{array}
$$

## The Language of Regrouping

"Don’t have enough ones"
"Regroup the tens
to ones"
"How many tens do

## 412 <br> - 175

I have left?"
"Becomes ten
ones"
"I now have twelve ones."
"I have no tens"
"I must regroup again"
"Regroup the hundreds to the tens"
"How many
hundreds do I have left?"
"Three hundreds, or three hundred"
"Becomes ten tens"
"I now have ten tens"
"Subtract seven tens from my ten tens"
"Three tens, or thirty"
"Three hundreds minus one hundred equals two hundreds"

## Long Division

## $9 \longdiv { 2 4 3 }$

## Number Bonds



## $8+5$

## Number Bonds

## $237+46$

## 53 minutes + 29 minutes

## Number Bonds

## 13 ounces + 8 ounces



## Number Bonds

## $35 \times 7$

## $468 \div 6$

## BAR MODELS

## 1. Read the Story

2. At the bottom of your work space, rewrite the question in the problem as a complete sentence leaving space for the answer.
3. Identify the "who" and "what" (units).
4. Draw a bar to model each variable.
5. Chunk the problem and adjust your unit bars to match your information. Fill in your question mark.
6. Work your computation; check.
7. Write the answer in your sentence; check.

## BAR MODELS: Explanations

1. Read the Story

- Do not do any math. Just read through and imagine the scene unfolding. Change names and take out numbers, if helpful.

2. At the bottom of your work space, rewrite the question in the problem as a complete sentence leaving space for the answer.
3. Identify the "who" and "what" (units).

- The units should be taken from your answer sentence.

4. Draw a bar to model each variable.

- Are you comparing two or more numbers?
- Begin with a start line
- Use one shorter bar for each number
- Make all bars equal - (you will adjust them to fit the story in step 5)
- Do you have a part-part-whole?
- Use one long bar
- If the answer to both these questions is no, try to draw a diagram of the situation.

5. Chunk the problem and adjust your unit bars to match your information. Fill in your question mark.

- Read the story one piece at a time, stopping when any new information is given that will help you fill in your bar model. For each number, ask: Is this all of the $\qquad$ or part of the $\qquad$ ?
- Once you have added information to your model, check your number (actually put a check mark over the number) and slash through the end of your phrase or sentence.

6. Work your computation; check.

- Show all work to the side of your model
- Check each step for accuracy

7. Write the answer in your sentence; check.

- Is your answer reasonable?


## Anatomy of a Bar Model:

ONE BAR:


TWO BARS:
(first whole)


## NOTES:

$\checkmark$ These are the two types you will see most often, although there will be other set-ups depending on the problem
$\checkmark$ Use straight lines (like a wide " $v$ ") if brackets are confusing
$\checkmark$ While textbooks and tests will use this format, feel free to have students put part numbers in the bars (I found this to be visually more helpful), then they can label the piece by its name or characteristic
$\checkmark$ Sometimes it is helpful to put the total at the end of the bar instead-if you choose to do this, be sure your students know how to label the ones in their books as well
$\checkmark$ Stress that when using fractions or multiples, units must be equal

## SPECIAL CIRCUMSTANCE MODELS:

## Fraction of a Fraction:



Example: Susan has a garden of roses. $1 / 3$ of her rose bushes are pink. $3 / 5$ of the remaining rose bushes are yellow. If she has 30 rose bushes in all, how many rose bushes are yellow?

Too many bars:


Example: Joshua bought 72 sheets of stickers. Each sheet has 8 stickers on it. How many stickers did Joshua buy?

## Model Drawing

## Pedro has 48 basketball trading cards. Joe gives him 18 more. How many basketball trading cards does Pedro have in all?

## Model Drawing

Daniel sells tickets to the fair to raise money. He sells 86 tickets on Monday. He sells 54 more tickets on Tuesday than on Monday. How many tickets does Daniel sell on Tuesday?

## Model Drawing

Anna has 68 flowers. She uses 56 flowers to make a garland. How many flowers does Anna have now?

## Model Drawing

## Ali scores 345 points in a card game. Farid scores 89 points less than Ali. How many points does Farid score?

## Model Drawing

## Christy had 4 times as many pencils as John. If they had 75 pencils altogether, how many pencils did Christy have?

## Model Drawing

Samantha is 6 years older than Jen. Abigail's age is 3 times Samantha's. The sum of Samantha and Jen's ages is 17 years less than Abigail's. How old is Jen?

